Supersymmetric Cosmology and Dark Energy

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Abstract: Using the superfield approach we construct the $n = 2$ supersymmetric lagrangian for the FRW Universe with perfect fluid as matter fields. The obtained supersymmetric algebra allowed us to take the square root of the Wheeler-DeWitt equation and solve the corresponding quantum constraint. This model leads to the relation between the vacuum energy density and the energy density of the dust matter.

Introduction

This paper is for the anniversary volume on the occasion 50th birthday, Sergei Odintsov, our colleague and friend who made an extensive contribution to the cosmological and astrophysics fields.

Some time ago we have used the superfield formulation to investigate supersymmetric cosmological models [1]. The main idea is to extend the group of local time reparametrization of the cosmological models to the local time supersymmetry which is a subgroup of the four dimensional space-time supersymmetry. This local supersymmetry procedure has the advantage that, by defining the superfields on superspace, all the component fields in a supermultiplet can be manipulated simultaneously in a manner that automatically preserves supersymmetry. Besides, the fermionic fields are obtained in a clear manner as the supersymmetric partners of the cosmological bosonic variables.

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More recently, using the superfield formulation the canonical procedure quantization for a closed FRW cosmological model filled with pressureless matter (dust) content and the corresponding superpartner was reported [2]. We have obtained the quantization for the energy-like parameter, and it was shown, that this energy is associated with the mass parameter quantization, and that such type of Universe has a quantized masses of the order of the Planck mass.

In the present work we are interested in the construction of the $n = 2$ supersymmetric lagrangian for the FRW Universe with barotropic perfect fluid as matter field including the cosmological constant. The simplest dark energy candidate is the cosmological constant stemming from energy density of the vacuum [3]. The obtained supersymmetric algebra allowed us to take the square root of the Wheeler-DeWitt equation and solve the corresponding quantum constraint.

Classical Action

The classical action for a pure gravity system and the corresponding term of matter content, perfect fluid with a constant equation of state parameter γ; $p = \gamma\rho$, and the cosmological term is [2]

$$S = \int \left[-\frac{c^2 R}{2N\tilde{G}} \left(\frac{dR}{dt}\right)^2 + \frac{Nkc^4}{2\tilde{G}}R + \frac{Nc^4\Lambda}{6\tilde{G}}R^3 + NM_\gamma c^2 R^{-3\gamma} \right] dt. \quad (1)$$

where c is the velocity of light in vacuum, $\tilde{G} = \frac{8\pi G}{6}$ where G is the Newtonian gravitational constant; $k = 1, 0, -1$ stands for spherical, plane or hyperspherical three space; $N(t), R(t)$ are the lapse function and the scale factor, respectively; M_γ is the mass by unit length$^{-\gamma}$.

The purpose of this work is the supersymmetrization of the full action (1) using the superfield approach. The action (1) is invariant under the time reparametrization

$$t' \rightarrow t + a(t), \quad (2)$$

if the transformations of $R(t)$ and $N(t)$ are defined as

$$\delta R = a\dot{R}, \qquad \delta N = (aN)^{\cdot} \quad (3)$$

The variation with respect to $R(t)$ and $N(t)$ lead to the classical equation for the scale factor $R(t)$ and the constraint, which generates the local reparametrization of $R(t)$ and $N(t)$. This constraint leads to the Wheeler-DeWitt equation in quantum cosmology.

In order to obtain the corresponding supersymmetric action for (1), we follow the superfield approach. For this, we extend the transformation of time reparametrization (2) to the $n = 2$ local supersymmetry of time $(t, \eta, \bar{\eta})$. Then, we have the following local supersymmetric transformation

$$\delta t = a(t) + \frac{i}{2}[\eta\beta'(t) + \bar{\eta}\bar{\beta}'(t)],$$

$$\delta\eta = \frac{1}{2}\bar{\beta}'(t) + \frac{1}{2}[\dot{a}(t) + ib(t)]\eta + \frac{i}{2}\dot{\bar{\beta}}'(t)\eta\bar{\eta}, \tag{4}$$

$$\delta\bar{\eta} = \frac{1}{2}\beta'(t) + \frac{1}{2}[\dot{a}(t) - ib(t)]\bar{\eta} - \frac{i}{2}\dot{\beta}'(t)\eta\bar{\eta},$$

where η is a complex odd parameter (η odd "time" coordinates), $\beta'(t) = N^{-1/2}\beta(t)$ is the Grassmann complex parameter of the local "small" $n = 2$ supersymmetry (SUSY) transformation, and $b(t)$ is the parameter of local $U(1)$ rotations of the complex η.

For the closed ($k = 1$) and plane ($k = 0$) FRW action we propose the following superfield generalization of the action (1), invariant under the $n = 2$ local supersymmetric transformation (4)

$$S_{susy} = \int \Big[-\frac{c^2}{2\tilde{G}} I\!\!N^{-1} I\!\!R D_{\bar{\eta}} I\!\!R D_{\eta} I\!\!R + \frac{c^3\sqrt{k}}{2\tilde{G}} I\!\!R^2 + \frac{c^3\Lambda^{1/2}}{3\sqrt{3}\tilde{G}} I\!\!R^3 -$$

$$- \frac{2\sqrt{2}M_\gamma^{1/2}}{(3 - 3\gamma)\tilde{G}^{1/2}} I\!\!R^{\frac{3-3\gamma}{2}} \Big] d\eta d\bar{\eta} dt, \tag{5}$$

where

$$D_\eta = \frac{\partial}{\partial\eta} + i\bar{\eta}\frac{\partial}{\partial t}, \qquad D_{\bar{\eta}} = -\frac{\partial}{\partial\bar{\eta}} - i\eta\frac{\partial}{\partial t}, \tag{6}$$

are the supercovariant derivatives of the global "small" supersymmetry of the generalized parameter corresponding to t. The local supercovariant derivatives have the form $\tilde{D}_\eta = I\!\!N^{-1/2}D_\eta$, $\tilde{D}_{\bar{\eta}} = I\!\!N^{-1/2}D_{\bar{\eta}}$, and $I\!\!R(t, \eta, \bar{\eta}), I\!\!N(t, \eta, \bar{\eta})$ are superfields.

The Taylor series expansion for the superfields $I\!\!N(t, \eta, \bar{\eta})$ and $I\!\!R(t, \eta, \bar{\eta})$ are the following

$$I\!\!N(t, \eta, \bar{\eta}) = N(t) + i\eta\bar{\psi}'(t) + i\bar{\eta}\psi'(t) + V'(t)\eta\bar{\eta}, \tag{7}$$

$$I\!\!R(t, \eta, \bar{\eta}) = R(t) + i\eta\bar{\lambda}'(t) + i\bar{\eta}\lambda'(t) + B'(t)\eta\bar{\eta}. \tag{8}$$

In the expressions (7) and (8) we have introduced the redefinitions $\psi'(t) = N^{1/2}\psi(t)$, $V' = N(t)V(t) + \bar{\psi}(t)\psi(t)$, $\lambda' = \frac{\tilde{G}^{1/2}N^{1/2}}{cR^{1/2}}\lambda$ and $B' = \frac{\tilde{G}^{1/2}}{c}NB + \frac{\tilde{G}^{1/2}}{2cR^{1/2}}(\bar{\psi}\lambda - \psi\bar{\lambda})$. The components of the superfield $I\!\!N(t, \eta, \bar{\eta})$ are gauge fields of the one-dimensional $n = 2$ extended supergravity. $N(t)$ is the einbein, $\psi(t), \bar{\psi}(t)$ are the complex gravitino fields, and $V(t)$ is the $U(1)$ gauge field. The component $B(t)$ in (8) is an auxiliary degree of freedom (non-dynamical variable), and $\lambda, \bar{\lambda}$ are the fermion partners of the scale factor $R(t)$. After the integration over the Grassmann coordinates $\theta, \bar{\theta}$ we can rewrite the action (5) in its component form

$$S_{susy} = \int \Big\{ -\frac{c^2R(DR)^2}{2N\tilde{G}} + \frac{i}{2}(\bar{\lambda}D\lambda - D\bar{\lambda}\lambda) - \frac{NR}{2}B^2 - \frac{N\tilde{G}^{1/2}B}{2cR}\bar{\lambda}\lambda +$$

$$+ \frac{c^2\sqrt{k}RN}{\tilde{G}^{1/2}}B + \frac{c^2\sqrt{k}R^{1/2}}{2\tilde{G}^{1/2}}(\bar{\psi}\lambda - \psi\bar{\lambda}) + \frac{cN\sqrt{k}}{R}\bar{\lambda}\lambda + \tag{9}$$

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$$+\frac{c^2\Lambda^{1/2}}{\sqrt{3}\tilde{G}^{1/2}}NR^2B+\frac{c^2\Lambda^{1/2}R^{3/2}}{2\sqrt{3}\tilde{G}^{1/2}}(\bar{\psi}\lambda-\psi\bar{\lambda})+\frac{2c\Lambda^{1/2}N}{\sqrt{3}}\bar{\lambda}\lambda-$$

$$-\sqrt{2}cM_\gamma^{1/2}NR^{\frac{1-3\gamma}{2}}B-\frac{\sqrt{2}}{2}cM_\gamma^{1/2}R^{-\frac{3\gamma}{2}}(\bar{\psi}\lambda-\psi\bar{\lambda})-$$

$$-\sqrt{2}(1-3\gamma)\tilde{G}^{1/2}M_\gamma^{1/2}NR^{\frac{-3-3\gamma}{2}}\bar{\lambda}\lambda\Big\}\,dt.$$

So, the lagrangian for the auxiliary field has the form

$$
\begin{aligned}
L_B &= -\frac{NR}{2}B^2 - \frac{N\tilde{G}^{1/2}B}{2cR}\bar{\lambda}\lambda + \frac{c^2\sqrt{k}RN}{\tilde{G}^{1/2}}B + \frac{c^2\Lambda^{1/2}NR^2}{\sqrt{3}\tilde{G}^{1/2}}B - \\
&\quad -\sqrt{2}cM_\gamma^{1/2}NR^{\frac{1-3\gamma}{2}}B.
\end{aligned}
\tag{10}
$$

From the expression (10) we can obtain the equation for the auxiliary field varying the Lagrangian with respect to B

$$B=\frac{c^2\sqrt{k}}{\tilde{G}^{1/2}}-\frac{\tilde{G}^{1/2}}{2cR^2}\bar{\lambda}\lambda+\frac{c^2\Lambda^{1/2}R}{\sqrt{3}\tilde{G}^{1/2}}-\sqrt{2}cM_\gamma^{1/2}R^{\frac{-3\gamma-1}{2}}. \tag{11}$$

Then, putting the expression (11) in (9) we have the following supersymmetric action

$$
\begin{aligned}
S_{susy} &= \int\Big\{ -\frac{c^2R(DR)^2}{2N\tilde{G}} + \frac{c^4NkR}{2\tilde{G}} + \frac{c^4N\Lambda R^3}{6\tilde{G}} + Nc^2M_\gamma R^{-3\gamma}+ \\
&\quad +\frac{c^4\sqrt{k}\Lambda^{1/2}R^2}{\sqrt{3}\tilde{G}} - \frac{\sqrt{2k}c^3}{\tilde{G}^{1/2}}M_\gamma^{1/2}R^{\frac{1-3\gamma}{2}} - \frac{\sqrt{2}c^3\Lambda^{1/2}M_\gamma^{1/2}}{\sqrt{3}\tilde{G}^{1/2}}R^{\frac{3-3\gamma}{2}}+ \\
&\quad +\frac{i}{2}(\bar{\lambda}D\lambda - D\bar{\lambda}\lambda) + \frac{cN\sqrt{k}}{2R}\bar{\lambda}\lambda + \frac{\sqrt{3}}{2}c\Lambda^{1/2}N\bar{\lambda}\lambda+ \\
&\quad +\frac{(-1+6\gamma)}{\sqrt{2}}N\tilde{G}^{1/2}M_\gamma^{1/2}R^{\frac{-3-3\gamma}{2}}\bar{\lambda}\lambda + \frac{c^2\sqrt{k}R^{1/2}}{2\tilde{G}^{1/2}}(\bar{\psi}\lambda-\psi\bar{\lambda}) \\
&\quad +\frac{c^2\Lambda^{1/2}}{2\sqrt{3}\tilde{G}^{1/2}}R^{3/2}(\bar{\psi}\lambda-\psi\bar{\lambda}) - \frac{\sqrt{2}}{2}cM_\gamma^{1/2}R^{-\frac{3\gamma}{2}}(\bar{\psi}\lambda-\psi\bar{\lambda})\Big\}\,dt,
\end{aligned}
\tag{12}
$$

where $DR=\dot{R}-\frac{i\tilde{G}^{1/2}}{2cR^{1/2}}(\psi\bar{\lambda}+\bar{\psi}\lambda)$ and $D\lambda=\dot{\lambda}-\frac{1}{2}V\lambda$, $D\bar{\lambda}=\dot{\bar{\lambda}}+\frac{1}{2}V\bar{\lambda}$.

Supersymmetric Quantum Model

In this section we will proceed with the quantization analysis of the system. The classical canonical Hamiltonian is calculated in the usual way for the systems with constraints. It has the form

$$H_c=NH+\frac{1}{2}\bar{\psi}S-\frac{1}{2}\psi\bar{S}+\frac{1}{2}VF, \tag{13}$$

where H is the Hamiltonian of the system, S and $\bar{S}$ are the supercharges and F is the $U(1)$ rotation generator. The form of the canonical Hamiltonian (13)

explains the fact that $N, \psi, \bar{\psi}$ and V are Lagrangian multipliers which only enforce the first-class constraints $H = 0, S = 0, \bar{S} = 0$ and $F = 0$, which express the invariance under the conformal $n = 2$ supersymmetric transformations. The first-class constraints may be obtained from the action (12) varying $N(t), \psi(t), \bar{\psi}(t)$ and $V(t)$, respectively. The first-class constraints are

$$
\begin{aligned}
H = {} & -\frac{\tilde{G}}{2c^2 R}\pi_R^2 - \frac{c^4 k R}{2\tilde{G}} - \frac{c^4 \Lambda R^3}{6\tilde{G}} - M_\gamma c^2 R^{-3\gamma} + \frac{\sqrt{2}c^3 \Lambda^{1/2} M_\gamma^{1/2}}{\sqrt{3}\tilde{G}^{1/2}}R^{\frac{3-3\gamma}{2}} - \\
& - \frac{c^4 \sqrt{k}\Lambda^{1/2}R^2}{\sqrt{3}\tilde{G}} + \frac{\sqrt{2k}c^3}{\tilde{G}^{1/2}}M_\gamma^{1/2}R^{\frac{1-3\gamma}{2}} - \frac{c\sqrt{k}}{2R}\bar{\lambda}\lambda - \frac{\sqrt{3}}{2}c\Lambda^{1/2}\bar{\lambda}\lambda - \\
& - \frac{(6\gamma - 1)}{\sqrt{2}}\tilde{G}^{1/2}M_\gamma^{1/2}R^{\frac{-3-3\gamma}{2}}\bar{\lambda}\lambda,
\end{aligned} \tag{14}
$$

$$
S = \left(\frac{i\tilde{G}^{1/2}}{cR^{1/2}}\pi_R - \frac{c^2\sqrt{k}R^{1/2}}{\tilde{G}^{1/2}} - \frac{c^2\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} + \sqrt{2}cM_\gamma^{1/2}R^{-\frac{3\gamma}{2}}\right)\lambda, \tag{15}
$$

$$
\bar{S} = \left(-\frac{i\tilde{G}^{1/2}}{cR^{1/2}}\pi_R - \frac{c^2\sqrt{k}R^{1/2}}{\tilde{G}^{1/2}} - \frac{c^2\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} + \sqrt{2}cM_\gamma^{1/2}R^{-\frac{3\gamma}{2}}\right)\bar{\lambda}, \tag{16}
$$

$$
F = -\bar{\lambda}\lambda, \tag{17}
$$

where $\pi_R = -\frac{c^2 R}{GN}\dot{R} + \frac{icR^{1/2}}{2N\tilde{G}^{1/2}}(\bar{\psi}\lambda + \psi\bar{\lambda})$ is the canonical momentum associated to R. The canonical Dirac brackets are defined as

$$
\{R, \pi_R\} = 1, \quad \{\lambda, \bar{\lambda}\} = i. \tag{18}
$$

With respect to these brackets the super-algebra for the generators $H, S, \bar{S}$ and F becomes

$$
\{S, \bar{S}\} = -2iH, \quad \{S, H\} = \{\bar{S}, H\} = 0, \quad \{F, S\} = iS, \quad \{F, \bar{S}\} = i\bar{S}. \tag{19}
$$

In a quantum theory the brackets (18) must be replaced by anticommutators and commutators, they can be considered as generators of the Clifford algebra. We have

$$
\{\lambda, \bar{\lambda}\} = -\hbar, \qquad [R, \pi_R] = i\hbar \qquad \text{with} \quad \pi_R = -i\hbar\frac{\partial}{\partial R} \tag{20}
$$

$$
\bar{\lambda} = \xi^{-1}\lambda^\dagger\xi = -\lambda^\dagger, \qquad \{\lambda, \lambda^\dagger\} = \hbar, \qquad \lambda^\dagger\xi = \xi\lambda^\dagger \quad \text{and} \quad \xi^\dagger = \xi.
$$

Then, for the operator $\bar{S}$ the following equation is satisfied

$$
\bar{S} = \xi^{-1}S^\dagger\xi. \tag{21}
$$

Therefore, the anticommutator of supercharges S and their conjugated operator $\bar{S}$ under our defined conjugation has the form

$$
\overline{\{S, \bar{S}\}} = \xi^{-1}\left\{S, \bar{S}\right\}\xi = \left\{S, \bar{S}\right\}, \tag{22}
$$

and the Hamiltonian operator is self-conjugated under the operation $\bar{H} = \xi^{-1}H^\dagger\xi$. We can choose the matrix representation for the fermionic parameters $\lambda, \bar{\lambda}$ and ξ as

$$
\lambda = \sqrt{\hbar}\sigma_-, \qquad \bar{\lambda} = -\sqrt{\hbar}\sigma_+, \qquad \xi = \sigma_3, \tag{23}
$$

with $\sigma_{\pm} = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$, where σ_1, σ_2, σ_3 are the Pauli matrices.

In the quantum level we must consider the nature of the Grassmann variables λ and $\bar{\lambda}$, with respect to these we perform the antisymmetrization, then we can write the bilinear combination in the form of the commutators, $\bar{\lambda}, \lambda \to \frac{1}{2}[\bar{\lambda}, \lambda]$, and this leads to the following quantum Hamiltonian H.

$$
\begin{aligned}
H_{quantum} &= -\frac{\tilde{G}}{2c^2}R^{-1/2}\pi_R R^{-1/2}\pi_R - \frac{c^4 kR}{2\tilde{G}} - \frac{c^4 \Lambda R^3}{6\tilde{G}} - M_\gamma c^2 R^{-3\gamma} \\
&+ \frac{\sqrt{2}c^3 \Lambda^{1/2} M_\gamma^{1/2}}{\sqrt{3}\tilde{G}^{1/2}} R^{\frac{3-3\gamma}{2}} - \frac{c^4 \sqrt{k}\Lambda^{1/2}R^2}{\sqrt{3}\tilde{G}} + \\
&+ \frac{\sqrt{2k}c^3}{\tilde{G}^{1/2}} M_\gamma^{1/2} R^{\frac{1-3\gamma}{2}} - \frac{c\sqrt{k}}{4R}[\bar{\lambda}, \lambda] - \frac{\sqrt{3}}{4}c\Lambda^{1/2}[\bar{\lambda}, \lambda] - \\
&- \frac{(6\gamma - 1)}{2\sqrt{2}} \tilde{G}^{1/2} M_\gamma^{1/2} R^{\frac{-3-3\gamma}{2}}[\bar{\lambda}, \lambda].
\end{aligned}
\tag{24}
$$

The supercharges S, $\bar{S}$ and the fermion number F have the following structures:

$$
S = A\lambda, \qquad S^\dagger = A^\dagger \lambda^\dagger \tag{25}
$$

where

$$
A = \frac{i\tilde{G}^{1/2}}{c}R^{-1/2}\pi_R - \frac{c^2\sqrt{k}}{\tilde{G}^{1/2}}R^{1/2} - \frac{c^2\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} + \sqrt{2}cM_\gamma^{1/2}R^{-\frac{3\gamma}{2}}, \tag{26}
$$

and

$$
F = -\frac{1}{2}[\bar{\lambda}, \lambda]. \tag{27}
$$

An ambiguity exist in the factor ordering of these operators, such ambiguities always arise, when the operator expression contains the product of non-commuting operator R and π_R, as in our case. It is then necessary to find some criteria to know which factor ordering should be selected. The inner product is calculated performing the integration with the measure $R^{1/2}dR$. With this measure the conjugate momentum π_R is non-Hermitian with $\pi_R^\dagger = R^{-1/2}\pi_R R^{1/2}$. However, the combination $(R^{-1/2}\pi_R)^\dagger = \pi_R^\dagger R^{-1/2} = R^{-1/2}\pi_R$ is a Hermitian one, and $(R^{-1/2}\pi_R R^{1/2}\pi_R)^\dagger = R^{-1/2}\pi_R R^{1/2}\pi_R$ is Hermitian too. This choice in our supersymmetric quantum approach $n = 2$ eliminates the factor ordering ambiguity by fixing the ordering parameter $p = \frac{1}{2}$.

Superquantum Solutions

In the quantum theory, the first-class constraints $H = 0, S = 0, \bar{S} = 0$ and $F = 0$ become conditions on the wave function $\Psi(R)$. Furthermore, any physical state must be satisfied the quantum constraints

$$
H\Psi(R) = 0, \quad S\Psi(R) = 0, \quad \bar{S}\Psi(R) = 0, \quad F\Psi(R) = 0, \tag{28}
$$

where the first equation is the Wheeler-DeWitt equation for the minisuperspace model. The eigenstates of the Hamiltonian (24) have two components in the matrix representation (23)

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}. \tag{29}$$

However, the supersymmetric physical states are obtained applying the supercharges operators $S\Psi = 0, \bar{S}\Psi = 0$. With the conformal algebra given by (19), these are rewritten in the following form

$$(\lambda\bar{S} - \bar{\lambda}S)\Psi = 0. \tag{30}$$

Using the matrix representation for λ and $\bar{\lambda}$ we obtain the following differential equations for $\Psi_1(R)$ and $\Psi_2(R)$ components

$$\left(\frac{\hbar\tilde{G}^{1/2}}{c}R^{-1/2}\frac{\partial}{\partial R} - \frac{c^2\sqrt{k}R^{1/2}}{\tilde{G}^{1/2}} - \frac{c^2\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} + \sqrt{2}cM_\gamma^{1/2}R^{-\frac{3\gamma}{2}} \right)\Psi_1(R) = 0. \tag{31}$$

$$\left(\frac{\hbar\tilde{G}^{1/2}}{c}R^{-1/2}\frac{\partial}{\partial R} + \frac{c^2\sqrt{k}R^{1/2}}{\tilde{G}^{1/2}} + \frac{c^2\Lambda^{1/2}R^{3/2}}{\sqrt{3}\tilde{G}^{1/2}} - \sqrt{2}cM_\gamma^{1/2}R^{-\frac{3\gamma}{2}} \right)\Psi_2(R) = 0. \tag{32}$$

Solving these equation, we have the following wave functions solutions

$$\Psi_1(R) = C\exp\left[\frac{\sqrt{k}c^3R^2}{2\hbar\tilde{G}} + \frac{c^3\Lambda^{1/2}}{3\sqrt{3}\hbar\tilde{G}}R^3 - \frac{2\sqrt{2}c^2M_\gamma^{1/2}}{(3-3\gamma)\hbar\tilde{G}^{1/2}}R^{\frac{3-3\gamma}{2}} \right], \tag{33}$$

$$\Psi_2(R) = \tilde{C}\exp\left[-\frac{\sqrt{k}c^3R^2}{2\hbar\tilde{G}} - \frac{c^3\Lambda^{1/2}}{3\sqrt{3}\hbar\tilde{G}}R^3 + \frac{2\sqrt{2}c^2M_\gamma^{1/2}}{(3-3\gamma)\hbar\tilde{G}^{1/2}}R^{\frac{3-3\gamma}{2}} \right]. \tag{34}$$

In the case of the flat universe ($k = 0$) and for the dust-like matter ($\gamma = 0$) we have the following solutions (using the relation $M_{\gamma=0} = \frac{1}{2}R^3\rho_{\gamma=0}$)

$$\Psi_1(R) = C_1\exp\left[\frac{1}{\sqrt{6\pi}}\left(\frac{\rho_\Lambda}{\rho_{pl}}\right)^{1/2}\left(\frac{R}{l_{pl}}\right)^3 - \frac{\sqrt{2}}{\sqrt{6\pi}}\left(\frac{\rho_{\gamma=0}}{\rho_{pl}}\right)^{1/2}\left(\frac{R}{l_{pl}}\right)^3 \right], \tag{35}$$

$$\Psi_2(R) = C_2\exp\left[-\frac{1}{\sqrt{6\pi}}\left(\frac{\rho_\Lambda}{\rho_{pl}}\right)^{1/2}\left(\frac{R}{l_{pl}}\right)^3 + \frac{\sqrt{2}}{\sqrt{6\pi}}\left(\frac{\rho_{\gamma=0}}{\rho_{pl}}\right)^{1/2}\left(\frac{R}{l_{pl}}\right)^3 \right], \tag{36}$$

where $\rho_{pl} = \frac{c^5}{\hbar G^2}$ is the Planck density and $l_{pl} = \left(\frac{\hbar G}{c^3}\right)^{1/2}$ is the Planck length.

We can see, that the function Ψ_1 in (35) has good behavior when $R \to \infty$ under the condition $\rho_\Lambda < 2\rho_{\gamma=0}$, while Ψ_2 does not. On the other hand, the wave function Ψ_2 in (36) has good behavior when $R \to \infty$ under the condition $\rho_\Lambda > 2\rho_{\gamma=0}$, because the principal contribution comes from the first term of the exponent, while Ψ_1 does not have good behavior. However, only the scalar product for the second wave function Ψ_2 is normalizable in the measure $R^{1/2}dR$ under the condition $\rho_\Lambda > 2\rho_{\gamma=0}$. This condition does not contradict the astrophysical observation at $\rho_\Lambda \approx (2-3)\rho_M$, due to the fact that the dust matter

introduces the main contribution to the total energy density of matter ρ_M.

On the other hand, according to recent astrophysical data, our universe is dominated by a mysterious form of the dark energy [4], which counts to about 70 per cent of the total energy density. As a result, the universe expansion is accelerating [5, 6]. Vacuum energy density $\rho_\Lambda = \frac{c^2 \Lambda}{8\pi G}$ is a concrete example of the dark energy.

Conclusion

The recent cosmological data give us the following range for the dark energy state parameter $\gamma = -0.96^{+0.08}_{-0.09}$. However, in the literature we can find different theoretical models for the dark energy with state parameter $\gamma > -1$ and $\gamma < -1$, see reviews [7, 8] and the articles [9, 10]. In the present work we have discussed the case for $\gamma = 0$ corresponding to the FRW universe with barotropic perfect fluid as matter field. In the case of the flat universe ($k = 0$) and the dust-like matter $\gamma = 0$ we have obtained two wave functions. However, only the second wave function is normalizable under the condition $\rho_\Lambda > 2\rho_{\gamma=0}$, which leads to the cosmological value $\Lambda > \frac{16\pi G}{c^2}\rho_{\gamma=0}$.

References

[1] O. Obregón, J.J. Rosales and V.I. Tkach, Phys. Rev. **D 53**, R1750 (1996); V.I. Tkach, J.J. Rosales and O. Obregón, Class. Quantum Grav. **13**, 2349 (1996).

[2] C. Ortiz, J.J. Rosales, J. Socorro, J. Torres and V.I. Tkach. Phys. Lett. **A 340**, 51-58, (2005); O. Obregón, J.J. Rosales, J. Socorro and V.I. Tkach, Class. Quantum Gravity, **16**, 2861 (1999); V.I. Tkach, J.J. Rosales and J. Socorro, Class. Quantum Gravity, **16**, 797 (1999); M. Ryan Jr, Hamiltonian Cosmology, Springer Verlag (1972); J. Socorro, M.A. Reyes and F.A. Gelbert, Phys. Lett. **A 313**, 338 (2003).

[3] V.I. Tkach, arXiv:0808.3429; C. Beck, arXiv:0810.0752.

[4] T. Padmanabhan, Phys. Rept. **380**, 235 (2003).

[5] S. Perlmutter, et al; Astrophysics J., **517**, 565 (1999).

[6] A.G. Riess, et al; Astrophysics J., **607**, 665 (2004).

[7] S. Nojiri, S.D. Odintsov, Int. J. Geom. Math. Mod. Phys. **4**, 115 (2007) and references therein.

[8] E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. **D** 15, 1753 (2006) and references therein.

[9] S. Nojiri, S.D. Odintsov, arXiv: 0801.4843; arXiv: 0807.0685; arxiv: 0810.1557.

[10] K. Bamba, C. Geng, S. Nojiri, S.D, Odinsov, arXiv:0810.4296